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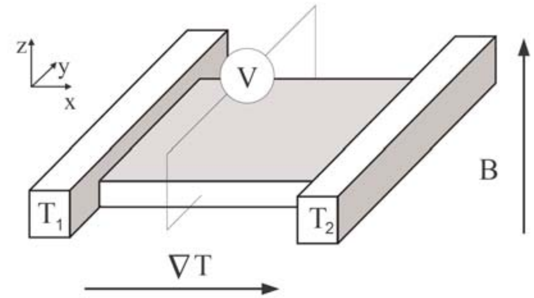
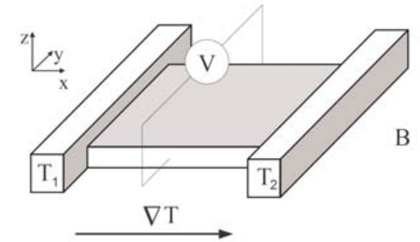


Fluctuations of the superconducting order parameter as an origin of the Nernst effect

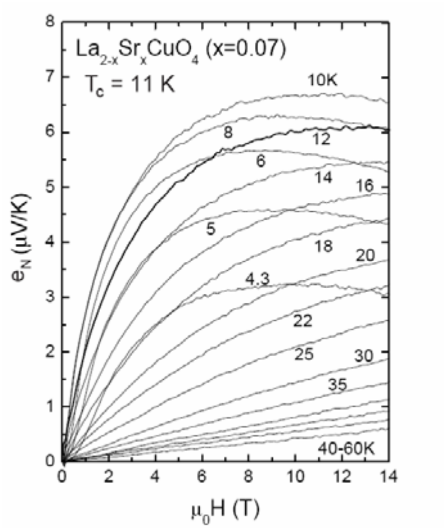
Karen Michaeli and Alexander M. Finkel’stein

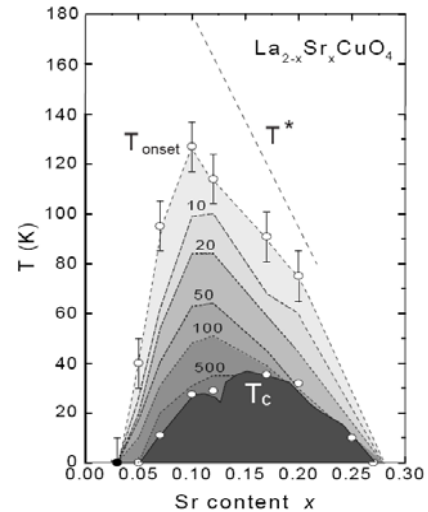
Nernst Effect- High Tc Materials



The Nernst signal

$$\nu = \frac{E_y}{-\nabla_x T \cdot B}$$

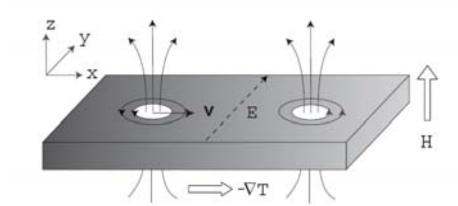




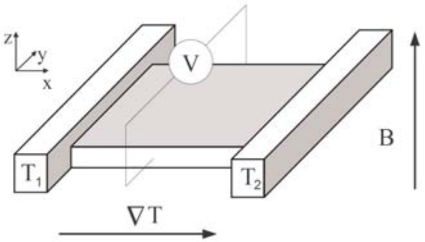
Y. Wang, *et al* 2005

Usually explained by the existence of vortices. → Pairing must survive above T_C. → Disappearance of phase coherence at T_C although the gap is still finite

Anderson, 2007
Raghu, et al, 2008
Mukerjee and Huse 2004



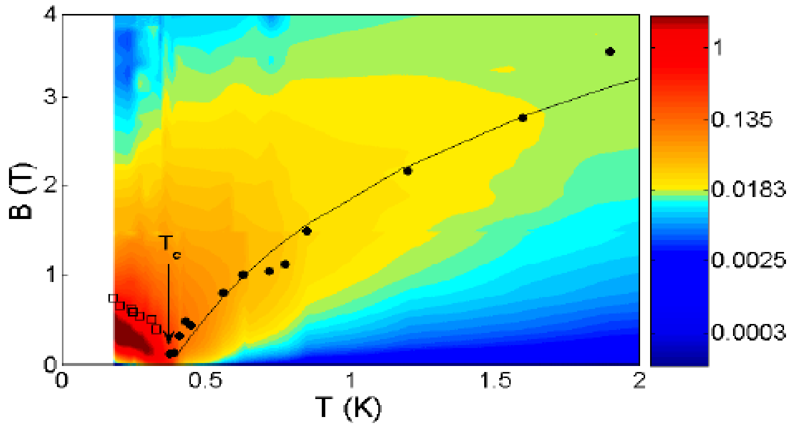
Nernst Effect – Conventional Superconductors



The Nernst signal

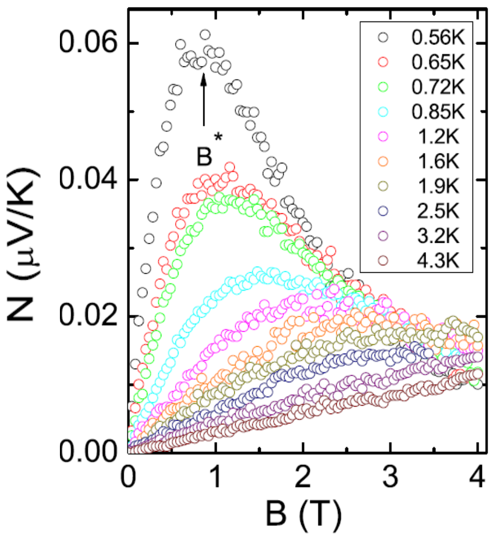
$$\nu = \frac{E_y}{-\nabla_x T \cdot B}$$

The strong Nernst signal above Tc can not be explained by the vortex-like fluctuations.



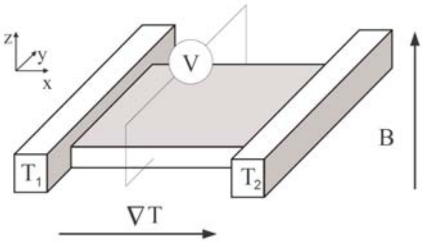
$Nb_{0.15}Si_{0.85}$

A. Pourret, *et al* 2007



It has been suggested that the fluctuations of the order parameter cause the effect.

Nernst Effect - Metals



The electric current in response to temperature gradient in a system with two species of particles (electrons and holes):

The Boltzamann equation for the distribution function:

$$\frac{\delta f_{e/h}(\mathbf{k})}{\tau} = \frac{\partial f_0(\varepsilon_\mathbf{k})}{\partial T}\mathbf{v}_\mathbf{k}\cdot\nabla T \mp \frac{e\mathbf{v}_\mathbf{k}}{c}\times\mathbf{B}\cdot\frac{\delta f_{e/h}(\varepsilon_\mathbf{k})}{\partial\mathbf{k}}$$

The electric current :

$$\mathbf{j}_e = -e\int\frac{d\mathbf{k}}{(2\pi)^d}\mathbf{v}_\mathbf{k}\delta f_e + e\int\frac{d\mathbf{k}}{(2\pi)^d}\mathbf{v}_\mathbf{k}\delta f_h$$

The longitudinal electric current:

$$j_e^x = e\int\frac{d\mathbf{k}}{(2\pi)^d}\frac{\partial f_0(\varepsilon_\mathbf{k})}{\partial\varepsilon_\mathbf{k}}\left[\frac{\varepsilon_\mathbf{k}v_\mathbf{k}^2\tau}{d} - \frac{\varepsilon_\mathbf{k}v_\mathbf{k}^2\tau}{d}\right]\frac{\nabla_x T}{T} = 0$$

The transverse electric current:

$$j_e^y = e\int\frac{d\mathbf{k}}{(2\pi)^d}\frac{\partial f_0(\varepsilon_\mathbf{k})}{\partial\varepsilon_\mathbf{k}}\frac{\varepsilon_\mathbf{k}v_\mathbf{k}^2\tau}{d}\frac{\nabla_x T}{T}\left[\omega_C\tau - (-\omega_C\tau)\right] \neq 0$$

$$\omega_C = \frac{eB}{mc}$$

Particle-hole symmetry does constrain the magnitude of the Nernst effect.

Under the approximation of a constant density of states:

$$j_e^y = 2e^2(\omega_C \tau)\frac{v_F^2 \tau}{d}\frac{\nabla_x T}{T}\int \frac{d\varepsilon_{\mathbf{k}}}{(2\pi)^d}\nu_0 \frac{\partial f_0(\varepsilon_{\mathbf{k}})}{\partial \varepsilon_{\mathbf{k}}}\varepsilon_{\mathbf{k}} = 0$$

For the collective modes the effective density of states is far from being a constant.

The neutral modes are not deflected by the Lorentz force.

The charged modes such as fluctuations of superconducting order parameter generate the Nernst effect.

→ Particle-hole symmetry does constrain the magnitude of the Nernst effect.

Under the approximation of a constant density of states:

$$j_e^y = 2e^2(\omega_C\tau)\frac{v_F^2\tau}{d}\frac{\nabla_x T}{T}\int\frac{d\varepsilon_{\mathbf{k}}}{(2\pi)^d}\nu_0\frac{\partial f_0(\varepsilon_{\mathbf{k}})}{\partial\varepsilon_{\mathbf{k}}}\varepsilon_{\mathbf{k}} = 0$$

The only source for the Nernst signal

The neutral modes are not deflected by the Lorentz force.

The charged modes such as fluctuations of superconducting order parameter contribute to the Nernst effect.

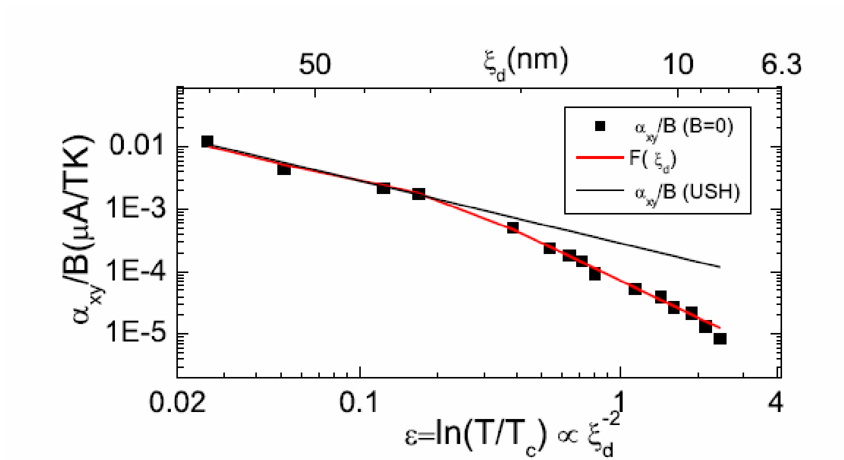
The Nernst Coefficient

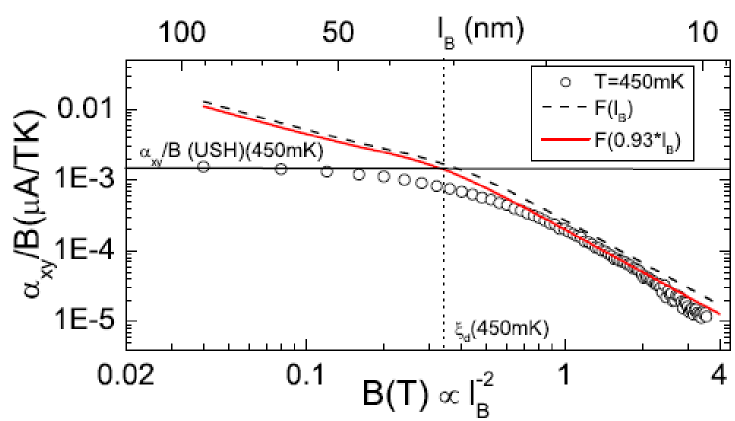
$$\begin{pmatrix} j \\ j_E \end{pmatrix} = \begin{pmatrix} \sigma & \alpha \\ \tilde{\alpha} & \kappa \end{pmatrix} \begin{pmatrix} E \\ \nabla T \end{pmatrix}$$

$$e_N = \frac{E_y}{-\nabla_x T} = \frac{\alpha_{xy}\sigma_{xx} - \alpha_{xx}\sigma_{xy}}{\sigma_{xy}^2 + \sigma_{xx}^2}$$

$$e_N \approx \frac{\alpha_{xy}}{\sigma_{xx}}$$

α_{xx} vanishes under the approximation of a constant density of states





A. Pourret, et al 2007

The Nernst Coefficient

The vertex of the temperature gradient is:

$$\nu(\varepsilon)\varepsilon\frac{\nabla T}{T}$$

Integrating over the frequency ε terms that in the electric conductivity are real becomes imaginary.

For example - the Aslamazov-Larkin term at B→0 lnT/Tc<<1 :

$$\mathbf{j}_e^{AL} \propto e\frac{\nabla T}{T}\eta\int\frac{d\mathbf{q}d\omega}{(2\pi)^{d+1}}\frac{\omega}{4\pi T}\frac{\partial n_P}{\partial\omega}\left[L_R(\mathbf{q},\omega)-L_A(\mathbf{q},\omega)\right]^2$$

L is the propagator of the fluctuations of the superconducting order parameter:

$$L_{R,A}=\frac{1}{\nu}\left[\ln\frac{T}{T_C}+\eta q^2+\chi\omega\mp\frac{i\pi\omega}{8T}\right]^{-1}$$

χ – is zero under the constant density of states approximation

Nernst Effect- Kubo Formula

Luttinger approach – J. M. Luttinger 1964.

Thermal conductivity:

- Introducing a gravitational field that is coupled to the Hamiltonian density:

$$H = \int h_0(r)dr + \int e^{-st}\gamma(r)h_0(r)dr$$

- Deriving the Kubo formula for the linear response to the field:

The E.O.M for the density matrix:

$$i\frac{d\rho(t)}{dt} = [H, \rho(t)] \qquad \dot{h} + \nabla j_E = 0$$

$$\longrightarrow \quad \langle j_E(r) \rangle = \langle e^{-\beta H_0} j_E j_E \rangle \nabla\gamma$$

Luttinger connected between the response to the gravitational field and the temperature gradient.

Magnetization

There has been a long discussion about the contribution of magnetization to the thermoelectric transport currents.

For example:
Obraztsov Sov. Phys. Solid State 1965
Smrcka and Streda J. Phys. C 1977
Cooper, Halperin and Ruzin PRB 1997

The heat current that describes the change in the entropy.

In the presence of magnetic field the thermodynamic expression for the heat contains the magnetization term:

$$dQ = TdS = dE - \mu dN + MdB.$$

The Kubo formula is not enough the contribution from the magnetization must be added.

Nernst Effect- Quantum Kinetic Equation

The Keldysh Green function: $\hat{G}(r,t;r't') = \begin{pmatrix} G_T & G^< \\ G^> & G_{\tilde{T}} \end{pmatrix} \Leftrightarrow \begin{pmatrix} G^R & G^K \\ & G^A \end{pmatrix}$

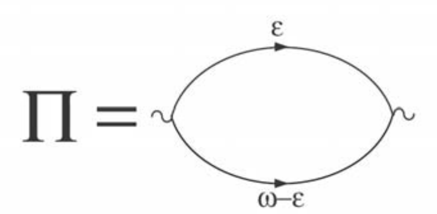
The current response to temperature gradient :

$$\mathbf{j}_e = i\left[e\hat{\mathbf{v}}_e(\nabla T)\hat{G}(\nabla T) + 2e\hat{\mathbf{v}}_\Delta(\nabla T)\hat{L}(\nabla T)\right]^<$$

The quasi-particles excitations

The fluctuations of the order parameter

$$\hat{L}(\mathbf{r}_1,t_1;\mathbf{r}',t') = \left[\lambda - \hat{\Pi}\right]^{-1}$$



v_e and v_Δ are the renormalized velocities

$\hat{G}(\nabla T;\mathbf{r}_1,t_1;\mathbf{r}',t')$ and $\hat{L}(\nabla T;\mathbf{r}_1,t_1;\mathbf{r}',t')$ are the solution of the quantum kinetic equation.

Nernst Effect- Magnetization

Magnetization

$$\hat{G}\left(\nabla T;\mathbf{R}=\frac{\mathbf{r}+\mathbf{r}'}{2},\mathbf{r}-\mathbf{r}';\varepsilon\right)$$

$$-\frac{\mathbf{R}\cdot\nabla T}{T}\varepsilon\frac{\partial\hat{G}_0(\mathbf{r}-\mathbf{r}';\varepsilon)}{\partial\varepsilon}$$

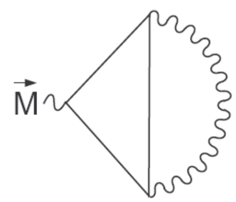
Local equilibrium part

$$\hat{G}(\nabla T;\mathbf{r}-\mathbf{r}';\varepsilon)$$

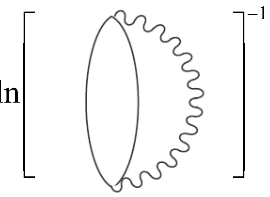
Translation invariant part

$$\hat{L}(\nabla T)=-L_0\Pi(\nabla T)L_0$$

$$j_e^y=ie\lim_{\mathbf{r}'\to\mathbf{r}}\int\frac{d\varepsilon}{2\pi}\frac{\nabla T}{T}MG_0^<(\mathbf{r}-\mathbf{r}';\varepsilon)$$



$-\partial/\partial B\ \ln$



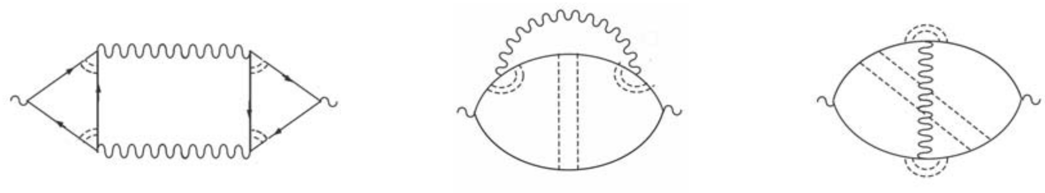
The Peltier coefficient is related to the flow of entropy

According to the third law of thermodynamics

$\alpha\to 0$ when $T\to 0$

The Peltier Coefficient

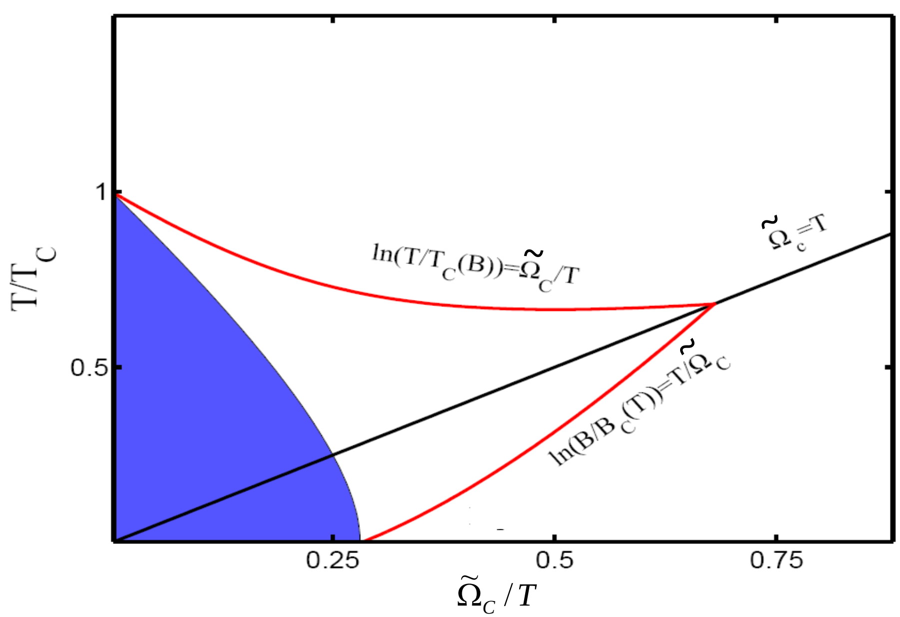
The contributing diagrams:



and the magnetization:

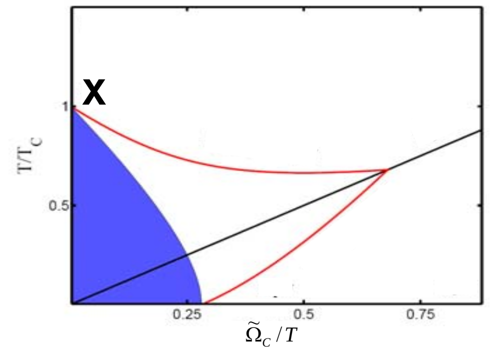
$$\alpha_{xy}^{mag} = -\frac{\partial}{\partial B}\frac{eB}{\pi}\sum_{N=0}^{\infty}\sum_{\omega_m}\ln\left\{-\nu\left[\ln\frac{T}{T_C}-\psi\left(\frac{1}{2}+\frac{|\omega_m|+\Omega_C(N+1/2)}{4\pi T}\right)-\psi\left(\frac{1}{2}\right)\right]\right\}$$

$$\Omega_c = \frac{4eDH}{c}$$



$$\tilde{\Omega}_C = \frac{\Omega_C}{4\pi}$$

The Peltier Coefficient



$$\Omega_C << T \qquad \ln \frac{T}{T_C} << 1$$

$$\alpha_{xy} \approx \frac{e\Omega_C}{192\,T \ln\left(T / T_C(B)\right)}$$

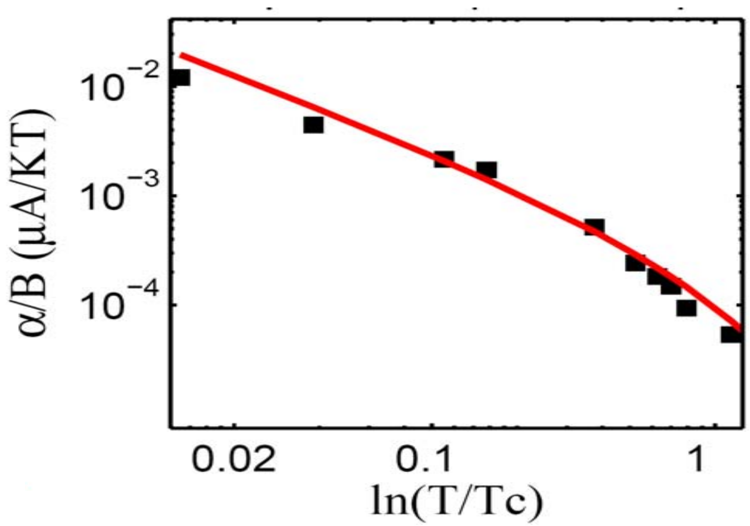
Classical fluctuations – coincide with the Phenomenological and microscopic result of Ussishkin et al, 2002 and Ussishkin 2003

Experimental data from A. Pourret, et al 2007

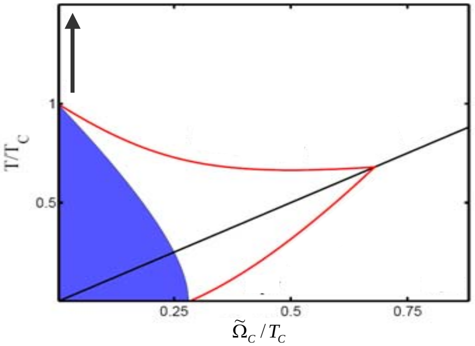
$Nb_{0.15}Si_{0.85}$ film of thickness $35\,nm$

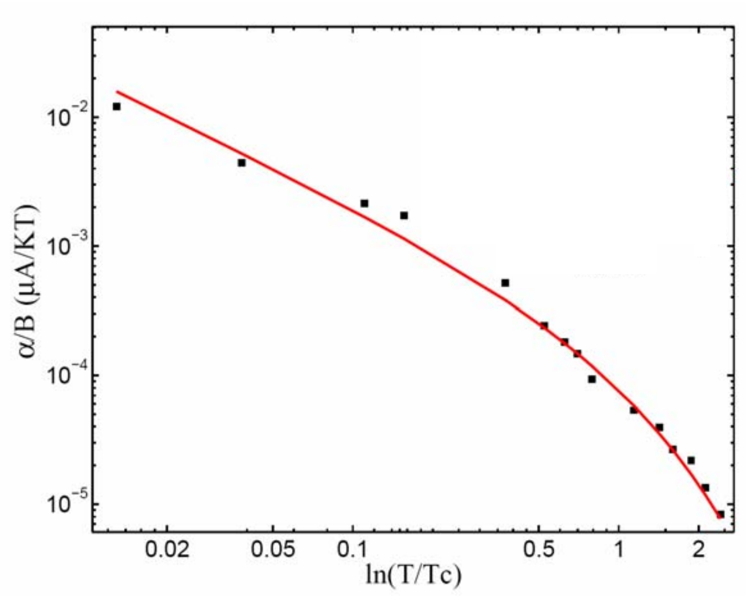
and $T_C = 380mK$

$$D = 0.187\, cm^2/\sec \qquad T_C^{MF} = 385mK$$



The Peltier Coefficient

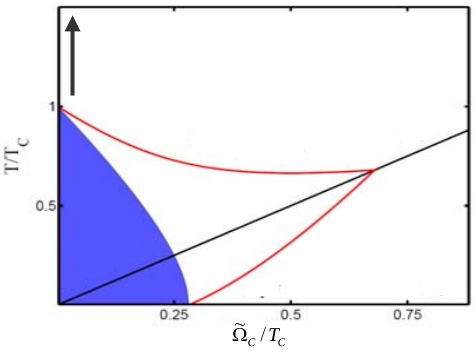


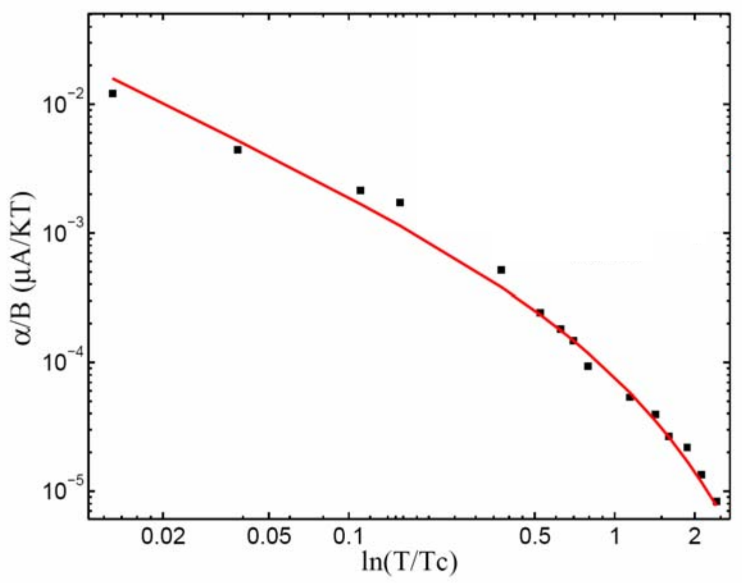


$$\Omega_C << T \qquad \ln\frac{T}{T_C} >> 1$$

$$\alpha_{xy} \approx \frac{e\Omega_C}{24\pi^2 T \ln\left(T / T_C\right)}$$

The Peltier Coefficient





$$\Omega_C << T \qquad \ln\frac{T}{T_C} >> 1$$

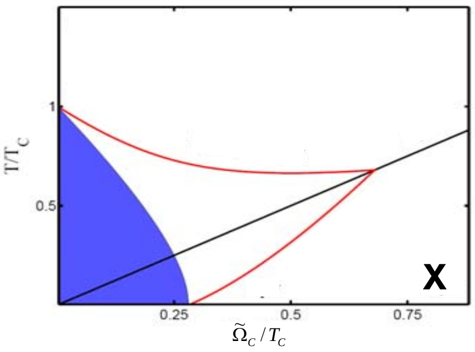
Quantum fluctuations – $T < \omega < 1/\tau$

The diagrams yield contributions of the order: $\ln\left(\ln\frac{1}{T\tau}\right) - \ln\left(\ln\frac{T}{T_C}\right)$

The logarithmically divergent terms are canceled out by the magnetization

Trace of the third law of thermodynamics

The Peltier Coefficient – High Magnetic field



$$\Omega_C >> T \qquad \ln\frac{B}{B_C} >> 1$$

The diagrams include contributions proportional to $\frac{\Omega_C}{T}$.

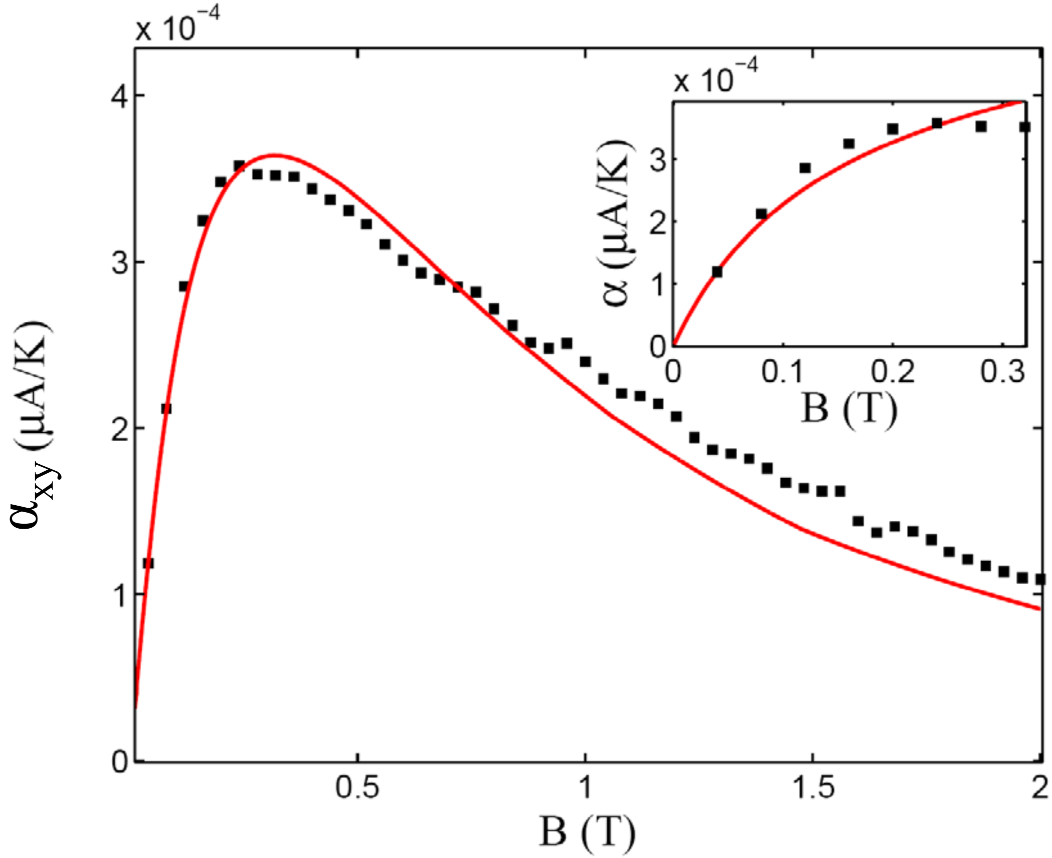
These terms are canceled out by the magnetization.

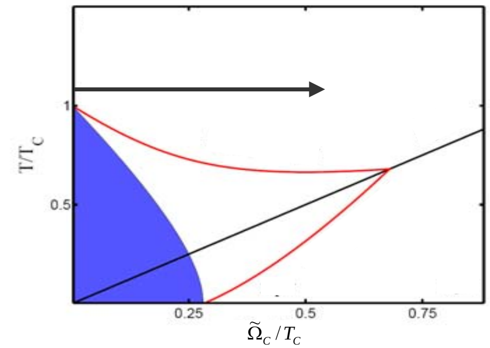
$$\alpha_{xy} \approx \frac{2eT}{3\Omega_C \ln\left(B/B_C\right)}$$

The Nernst signal goes to zero at T→0.

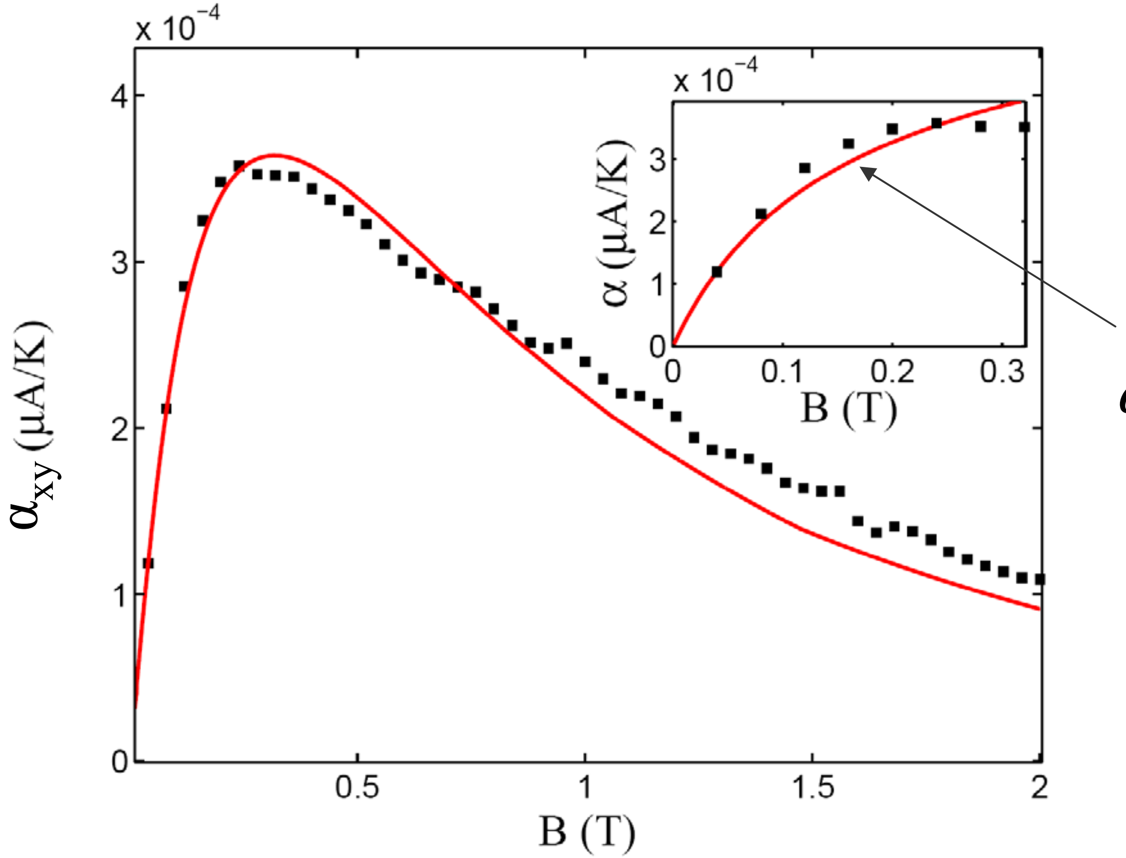
Consistent with the third law of thermodynamics.

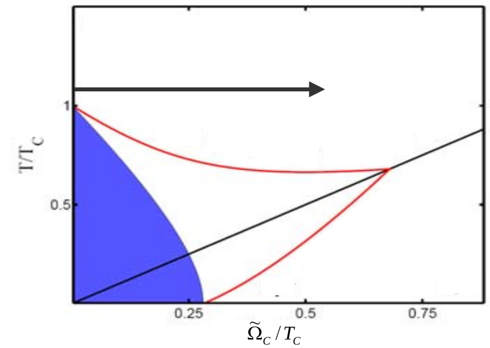
The Peltier Coefficient as a Function of the Magnetic Field





The Peltier Coefficient as a Function of the Magnetic Field





$$\alpha_{xy} \approx \frac{e\Omega_C}{192\,T\,\ln\left(T/T_C(B)\right)}$$

Summary

- The contribution from the fluctuations of the superconducting order parameter to the Nernst effect is dominant and can be observed far away from the transition.
- The important role of the magnetization is in canceling the quantum contributions, thus making the Nernst signal compatible with the third law of thermodynamics.
- As a consequence of the constrain imposed by the third law of thermodynamics, the phase diagram is less rich and diverse than one expects in the vicinity of a quantum phase transition.
- The Nernst effect provides an excellent opportunity to test the use of the quantum kinetic equation in the description of thermoelectric transport phenomena.
- Our results are different in few aspects from the expressions for the Peltier coefficient recently obtained using the Kubo formula by Serbyn, Skvortsov, Varlamov, and Galitski.